

## STOR 654 Homework 6

1. Establish the following facts about the conditional expectation for discrete random variables  $X$  and  $Y$ .

(a) Law of total expectation,  $\mathbb{E}\{\mathbb{E}(Y|X)\} = \mathbb{E}Y$

(b) Functions of  $X$  act like constants,  $\mathbb{E}[f(X)g(Y)|X] = f(X)\mathbb{E}(g(Y)|X)$

(c) Jensen's Inequality. If  $g : \mathbb{R} \rightarrow \mathbb{R}$  is convex then  $g(\mathbb{E}(Y|X)) \leq \mathbb{E}(g(Y)|X)$

2. Let  $X$ ,  $Y$ , and  $Z$  be random variables defined on the same probability space, and assume that  $\mathbb{E}X^2$  and  $\mathbb{E}Y^2$  are finite. Define the conditional covariance of  $Y$  and  $Y$  given  $Z$  by

$$\text{Cov}(X, Y | Z) = \mathbb{E}(XY | Z) - \mathbb{E}(X | Z)\mathbb{E}(Y | Z).$$

Note that the conditional covariance is a random variable and can be expressed as a function of  $Z$ . Use conditioning arguments to establish the following identity, sometimes called the law of total covariance

$$\text{Cov}(X, Y) = \mathbb{E}(\text{Cov}(X, Y | Z)) + \text{Cov}(\mathbb{E}(X | Z), \mathbb{E}(Y | Z)).$$