STOR 654 Homework 4

1. Let $\mathcal{P}$ be an exponential family generated by $T : \mathcal{X} \to \mathbb{R}^d$ and $h(x) : \mathcal{X} \to [0, \infty)$ with parameters $\eta \in \mathbb{R}^d$. Show that the maximum likelihood estimator of η coincides with the method of moments estimator.

2. Let $\mathcal{D}$ be a family of decision rules for a decision problem. Show that if $d \in \mathcal{D}$ is admissible and has constant risk then it is minimax.

3. (Ancillary Statistics). Let $\mathcal{P} = \{f(x|\theta) : \theta \in \Theta\}$ be a family of distributions on a sample space, and let $X \in \mathcal{X}$ have density $f(x|\theta) \in \mathcal{P}$. A statistic $T : \mathcal{X} \to \mathcal{T}$ is said to be *ancillary* if the distribution of $T(X)$ does not depend is the same for each $\theta \in \Theta$.

   (a) Show that if $X_1, \ldots, X_n$ are i.i.d. samples from a location family $\mathcal{P} = \{f(x-\theta) : \theta \in \mathbb{R}\}$ then $T(x) = x_{(n)} - x_{(1)}$ is ancillary.

   (b) Show that if $X_1, \ldots, X_n$ are i.i.d. samples from a scale family $\mathcal{P} = \{\sigma^{-1} f(\sigma x) : \sigma > 0\}$ then $T(x) = x_1/(x_1 + x_2)$ is ancillary.