STOR 655 Homework 7

1. Let $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ be positive constants.

   a. Use Jensen's inequality to establish the Arithmetic-Geometric mean inequality

   $$\frac{1}{n}\sum_{i=1}^{n} a_i \ \geq \ \left(\prod_{i=1}^{n} a_i\right)^{1/n}.$$

   b. Establish the inequality

   $$(\Pi_{k=1}^{n} a_k)^{1/n} \ + \ (\Pi_{k=1}^{n} b_k)^{1/n} \ \leq \ (\Pi_{k=1}^{n}(a_k + b_k))^{1/n}$$

   Hint: First divide the LHS by the RHS.

2. Let X be a non-negative random variable such that $\mathbb{E}X^2$ is finite. Show that for each $0 < \lambda < 1$ we have the inequality

$$\mathbb{P}(X \geq \lambda\,\mathbb{E}X) \ \geq \ (1-\lambda)^2 \, \frac{(\mathbb{E}X)^2}{\mathbb{E}X^2}$$

Hint: Use the Cauchy-Schwartz inequality and the identity $X = X\,\mathbb{I}(X \geq c) + X\,\mathbb{I}(X < c)$.