STOR 215: Sample Questions for Final

Here are some sample questions for the final exam (questions on the actual final will be different). Remember that the final exam is comprehensive: these questions only concern the material on graphs

All exams in the course are closed book and closed notes, and calculators are not permitted. Most questions will have multiple parts, and in many cases different parts of the same question are unrelated, so if you cannot solve one part of a question, you should still try the other parts.

1. Let $G = (V, E)$ be an undirected graph.

   - Define cut vertex.
   - When is G biparatite?

2. Carefully define what it means for two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ to be isomorphic.

   - Define the notion of an isomorphism invariant
   - Given an example of an isomorphism invariant

3. Draw the graphs $K_{2,2}$, C_4, and K_3. Find the chromatic number of each graph.

4. An undirected graph G has the following adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   - Draw the graph G. Is G connected?
   - Does G have a cut vertex?
   - Does G have a Hamilton path?

5. Let G be a simple graph and let u be a vertex in G. Show that if there is a circuit c from u to u then there is a simple circuit from u to u.

6. A connected, simple, planar graph G with 9 vertices divides the plane into 3 regions. How many edges does G have?

7. Let $G = (V, E)$ be a bipartite graph with vertex partition $V = V_1 \cup V_2$. Hall's theorem gives necessary and sufficient conditions for the existence of a complete matching from V_1 to V_2. Carefully state these conditions.

8. A simple graph $G = (V, E)$ has four vertices. The following are possible sequences of degrees for the vertices in V. In each case draw a graph with the given degree sequence or say why no such simple graph exists.

a. 1, 1, 0, 0

b. 2, 1, 0, 0

c. 4, 1, 1, 0