STOR 215: Sample Questions for Midterm 2

Here are some sample questions for the second midterm exam. Questions on the actual midterm will be different. Remember that all exams in the course are closed book and closed notes, and calculators are not permitted. Most questions will have multiple parts, and in many cases different parts of the same question are unrelated, so if you cannot solve one part of a question, you should still try the other parts.

1. Find a closed form expression for the sequence defined by the recurrence relations $a_0 = 3$ and $a_{n+1} = a_n + 2$ for $n \geq 0$.

2. Show that if $a \equiv b \bmod m$ and $c \equiv d \bmod m$ then $ac \equiv bd \bmod m$.

3. Give a simple numerical expression for $\sum_{k=0}^{n} \binom{n}{k}$.

4. In each of the following cases, indicate whether the set described is finite or infinite. You need not prove anything.

a. The rational numbers between 1 and 3.

b. The real numbers between 1 and 3.

b. The set of real numbers x such that $12x + 4 = 5$.

c. The set of real numbers $\{1/n : n \geq 1\}$.

5. Audrey has a large jar filled with dimes, nickels, and quarters. In how many ways can she leave home with 10 coins from the jar under the following conditions?

a. There are no constraints.

b. She takes at least 2 dimes, 2 nickels, and 3 quarters.

6. Find the numerical value of

a. $\sum_{n=1}^{3} \lfloor \sqrt{n} \rfloor$

b. $\sum_{j=1}^{8} j$

c. $\sum_{i=1}^{3} \sum_{j=1}^{2} (-1)^{i+j}$

7. Carefully describe the setting and procedure for proof by (weak) induction.

8. A company has 850 employees who were born in the United States. What can you say about the following:

a. The number of employees born in the same state?

b. The number of employees born in Arizona?

c. The number of employees sharing the same last two digits of their Social Security number?

9. A board game ends with a win (W), tie (T), or loss (L). Suppose that you play the game 6 times in a row. Let an outcome be the record of your wins, losses, and ties over the 6 plays, for example, WLLTWL or LLTWTT.

a. How many possible outcomes are there for the 6 plays?

b. How many outcomes begin with WWL?

c. How many outcomes begin with two wins and end with two losses?

d. How many outcomes begin with two wins or end with a loss?

10. Find the following.

a. The prime factorization of 390.

b. $\gcd(55, 231)$

c. 53 mod 6

d. -87 mod 11

11. A high school athlete heading to college wishes to give away 20 different trophies to 3 family members. If she intends to give 5 trophies to the first family member, 5 to the second, and 10 to the third, how many ways can she give the trophies away?