STOR 215: Sample Questions for Midterm 1

Here are some sample questions for the first midterm exam. Questions on the actual midterm will be different. Remember that all exams in the course are closed book and closed notes, and calculators are not permitted. Most questions will have multiple parts, and in many cases different parts of the same question are unrelated, so if you cannot solve one part of a question, you should still try the other parts.

1.a. Construct a truth table for the proposition $p \to q$

b. Define what it means for a compound proposition p to be a tautology.

c. Find the Cartesian product of the sets $\{\text{Don, Sue}\}$ and $\{P, F\}$ and its cardinality.

d. Find the following: $\lfloor -7/6 \rfloor =$ $\qquad \lfloor 3/8 \rfloor =$ $\qquad \lceil -3.25 \rceil =$

e. Let $f : A \to B$ be a function. For $S \subseteq A$ carefully define the image $f(S)$.

2. Give the truth value of the following propositions concerning the real numbers $\mathbb{R}$.

a. $2^2 = 4 \to 3^2 = 5$

b. $(\exists x \; x^2 = x + x) \vee 3 + 4 = 7$

c. $\forall x \, (x^2 \geq 0 \wedge \exists y \; x < y)$

d. $(\forall x \; x > 3) \to 1 + 2 = 4$

3. Express the negations of the following propositions so that no negation appears outside a quantifier or an expression involving a logical operation. Your final answers should not involve the implication ($\to$) operation.

a. $\forall x \, (P(x) \vee Q(x))$

b. $\exists x \, \forall y \, (P(x, y) \to P(y, x))$

4. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 2$ and $g(x) = 3x - 1$. Express each of the following as a function of x. Present your answer in the simplest possible form.

a. $f + g$

b. $f \circ g$

5. Let p, q, r be the following propositions:

$p =$ Bob is a statistician $\quad q =$ Bob likes numbers $\quad r =$ Bob is fun at parties

Translate each of the following sentences into a logical expression involving p, q, and r.

a. Bob likes numbers and he is a statistician.

b. Although Bob likes numbers and he is a statistician, he is also fun at parties.

c. Bob likes numbers but he is not a statistician.

d. For Bob to be a statistician, it is necessary for him to like numbers and be boring at parties.

e. Bob is a statistician if and only if he likes numbers and is fun at parties.

6. Consider the following functions $f : \mathbb{R} \to \mathbb{R}$. In each case, say whether the function is (i) one-to-one, (ii) onto, a (iii) bijection. Briefly explain your answer.

a. $f(x) = 2x - 1$

b. $f(x) = 3 \sin x$

c. $f(x) = 1 - x$

7. Let $A = \{a, b, c\}$, $B = \{b, c, d, e\}$, and $C = \{c\}$ be subsets of a universal set $U = \{a, b, c, d, e, f\}$.

Identify the following.

a. $A \cap B$

b. $B \setminus C$

c. $|A|$

Indicate whether each of the following relations is true or false.

d. $\{\emptyset\} \in A$

e. $\{b, c\} \subseteq A$

f. $B \subseteq 2^B$

8. Consider a group of climbers that climb together on weekends. Let the predicate $C(x)$ denote that x has a climbing certificate, and let $A(x, y)$ denote that climber x has assisted climber y. Translate each of the following statements into a logical expression involving $C(x), A(x, y)$ and quantifiers. Let the universal set U be the set of climbers attending the monthly hikes.

a. Sylvia has a certificate.

b. Amy does not have a certificate, but has assisted Todd.

c. Rob has not assisted Sandra.

d. No one has assisted Joseph.

e. Everyone with a certificate has assisted another climber.

9. Establish the each of the logical equivalences below in a step by step fashion using the rules of propositional equivalence from the text. At each step, you should either identify the rule used to obtain that step by name (if that rule appears on the study list), or provide the general form of the rule. You need not indicate uses of the commutative or distributive laws.

a. $p \to q \equiv \neg q \to \neg p$

b. $(p \vee q) \wedge \neg(p \wedge q) \equiv (p \wedge \neg q) \vee (q \wedge \neg p)$

10. Show that for integers x, $3x + 2$ is even if and only if $x + 5$ is odd. Provide a clear argument using English as appropriate.