Statistics 215 Homework 3

1. Find the following

   a. $\max(34, -256)$

   b. $\min(-34, -256)$

   c. $\max(-128, -2)$

   d. $\max(0, \min(-20, -5))$

   e. $\min(3, \max(-2, -5))$

2. Show that $\min(a, b) \leq a, b$.

3. Show that $\min(a, b) + \max(a, b) = a + b$.

4. Show that $|x| = \max(x, -x)$.

5. The triangle inequality states that $|x + y| \leq |x| + |y|$.

   a. Establish the triangle inequality by considering cases involving the signs of x and y.

   b. Show that the triangle inequality implies $|x - y| \leq |x - z| + |z - y|$.

   c. The quantity $|x-y|$ measures the distance between x and y. What does the inequality in part (b) say about distances between points?

6. Let $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ be real numbers. Consider the following argument concerning the maximum of $(a_i + b_i)$. By definition,

$$\max_{1 \leq i \leq n} (a_i + b_i) = a_r + b_r$$

for some r in the set $\{1, \ldots, n\}$. Moreover, for this r,

$$a_r \leq \max_{1 \leq j \leq n} a_j \quad \text{and} \quad b_r \leq \max_{1 \leq k \leq n} b_k.$$

Putting these inequalities together using the rule $a \leq b \rightarrow a + c \leq b + c$ we obtain the inequality

$$\max_{1 \leq i \leq n} (a_i + b_i) \;=\; a_r + b_r \;\leq\; \max_{1 \leq j \leq n} a_j + \max_{1 \leq k \leq n} b_k.$$

Use a similar argument to find an inequality relating the minima

$$\min_{1 \leq i \leq n} (a_i + b_i) \quad \text{and} \quad \min_{1 \leq j \leq n} a_j + \min_{1 \leq k \leq n} b_k.$$

7. Show that if $a_1 \leq a_2$ and $b_1 \leq b_2$ then $\max(a_1, b_1) \leq \max(a_2, b_2)$. Hint: you may wish to consider the cases $\max(a_1, b_1) = a_1$ and $\max(a_1, b_1) = b_1$.

8. Use logical arguments (e.g., logical identities, equivalences) to establish the following set identities

(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(c) $\overline{A \cap B} = \overline{A} \cup \overline{B}$