Statistics 215 Homework 2

1. Show that if $a \le b$ and $b \le c$ then $a \le c$. (Repeat the add-and-subtract argument from class.)

2. Establish whether the following propositions are true or false. When quantifiers are present, the domain is the real numbers.

   a. $4 \le -3$

   b. $\exists x \; x^2 = x$

   c. $\forall x \; x^2 \ge x$

   d. $\forall x \, \forall y \; y^2 + x \ge x$

   e. $\exists x \; x > x^3$

   e. $\forall x \, \forall y \; (x \ge 0 \wedge y \le 0) \to (x \, y \ge 0)$

3. Show that if $0 \le a \le b$ and $0 \le c \le d$ then $ac \le bd$. (Repeat the add-and-subtract argument from class.) Show by example that the implication fails to hold if we do not require that all numbers be non-negative.

4. Establish the following by direct argument from the definition.

   a. If $a \le 0$ then $a + c \le c$ for every c

   b. If $a \le b$ then $a + c \le b + c$ for every c

   c. If $\alpha \le 0$ and $a \le b$ then $\alpha b \le \alpha a$